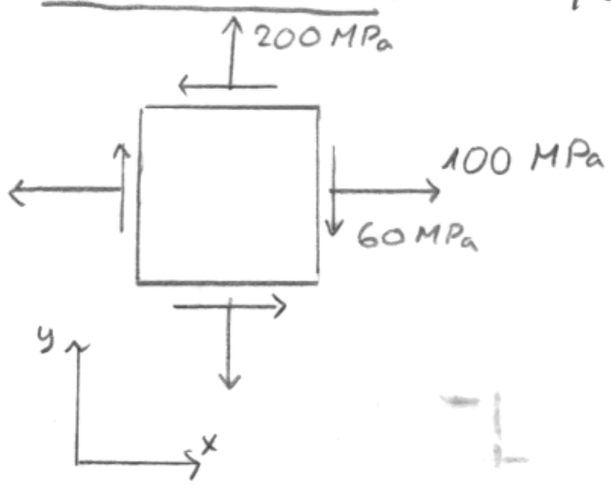
ZADANIE DOMOWE z WK I - seria I 2020 r.

Maciej Jasiński

I = 6
N = 8

Zadanie 1:

P.S.N.

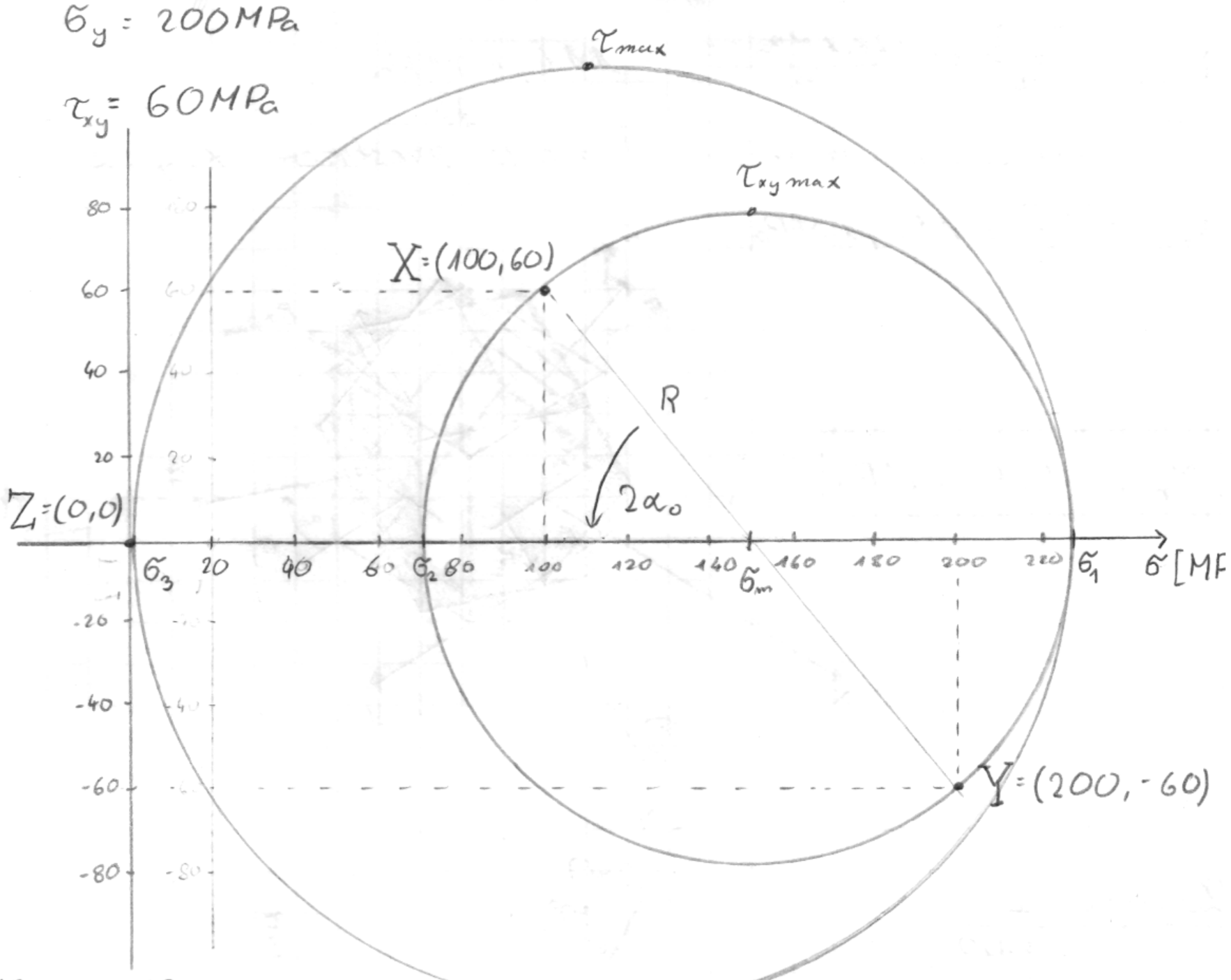


1 | 1
2 | 1
3 | 1

$\sigma_x = 100\,MPa$

$\sigma_y = 200\,MPa$

$\tau_{xy} = 60\,MPa$



$$\sigma_m = \frac{\sigma_x + \sigma_y}{2} = \frac{100\,MPa + 200\,MPa}{2} = 150\,MPa$$

$$R = \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{200\,MPa - 100\,MPa}{2}\right)^2 + (60\,MPa)^2} \approx 78,10\,MPa$$

- Wartości główne naprężenia:

$\underline{\sigma_1} = \sigma_m + R = 150\,MPa + 78,10\,MPa \approx \underline{228,1\,MPa}$

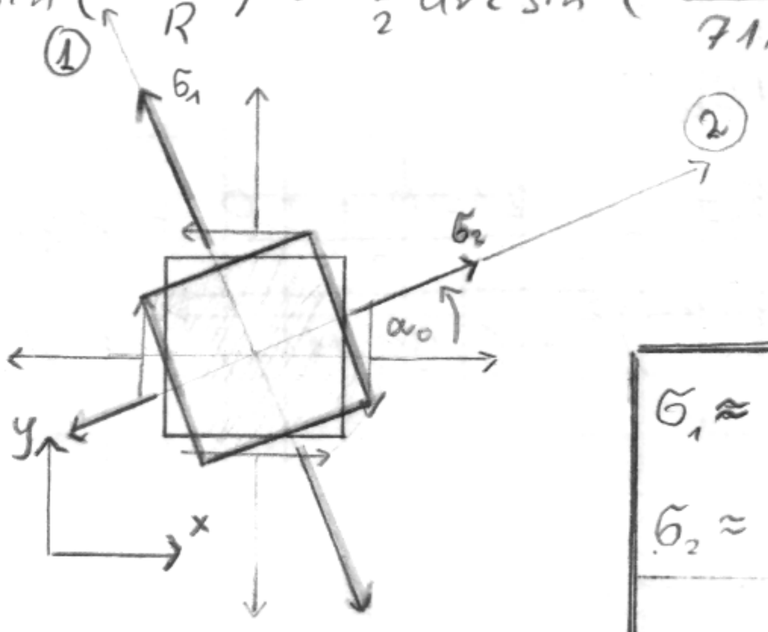
$\underline{\sigma_2} = \sigma_m - R = 150\,MPa - 78,10\,MPa \approx \underline{71,90\,MPa}$

- Wartość maksymalnych naprężeń tnących na płaszczyźnie x,y:

$\underline{\tau_{(xy)max}} = R \approx \underline{78,10\,MPa}$

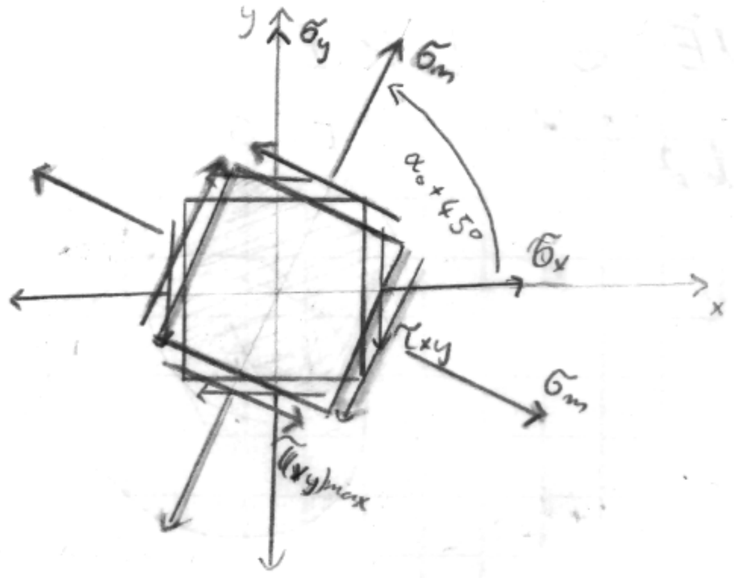
- Kierunek główny:

$\underline{\alpha_0} = \frac{1}{2}\arcsin\left(\frac{\tau_{xy}}{R}\right) = \frac{1}{2}\arcsin\left(\frac{60\,MPa}{71,90\,MPa}\right) \approx \underline{25,10^\circ}$



nie jest to maksymalna wartość naprężeń tnących biorąc pod uwagę przestrzeń.

$\sigma_1 \approx 228,1\,MPa$
$\sigma_2 \approx 71,90\,MPa$
$\alpha_0 \approx 25,10^\circ$



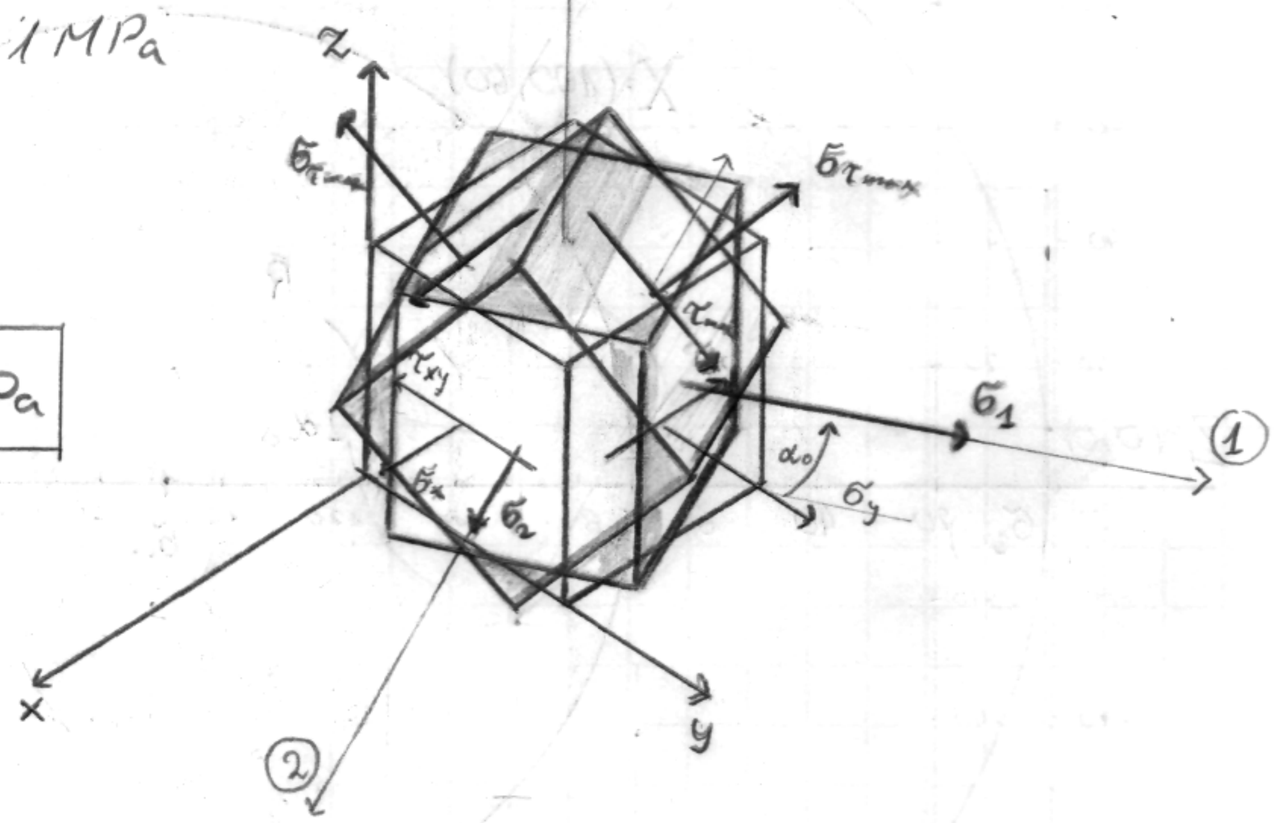
$\sigma_m = 150\ MPa$

$\tau_{(xy)max} \approx 78{,}10\ MPa$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \approx \frac{228{,}1\ MPa - 0}{2} \approx 114{,}1\ MPa$$

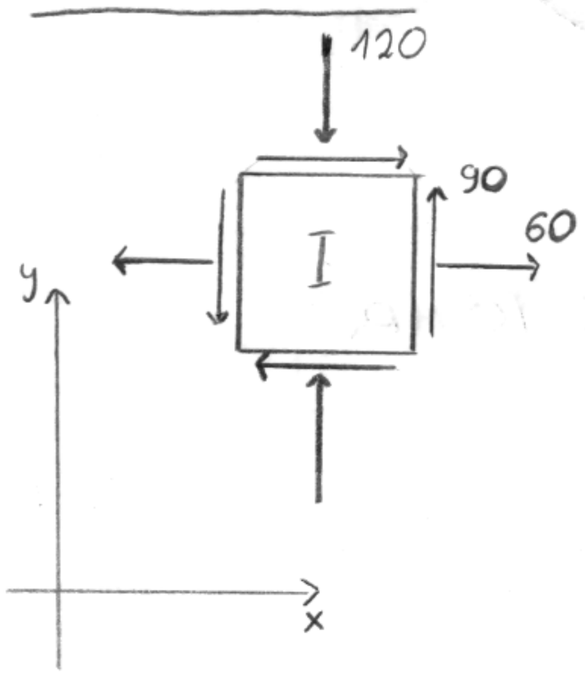
τ maksymalne naprężenie tnące w przestrzeni x, y, z

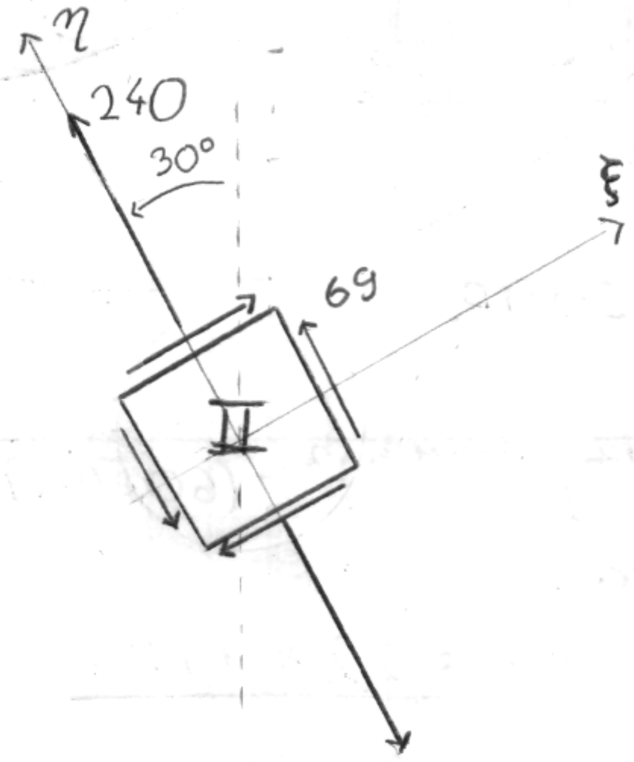
$$\sigma_{\tau max} = \frac{\sigma_1 + \sigma_3}{2} = 114{,}1\ MPa$$



$\tau_{max} = 114{,}1\ MPa$

Zadanie 2:





$\sigma_{xI} = 60\ MPa$ $\qquad$ $\sigma_{\eta II} = 240\ MPa$

$\sigma_{yI} = -120\ MPa$ $\qquad$ $\sigma_{\xi II} = 0$

$\tau_{xyI} = -90\ MPa$ $\qquad$ $\tau_{\xi\eta II} = -69\ MPa$

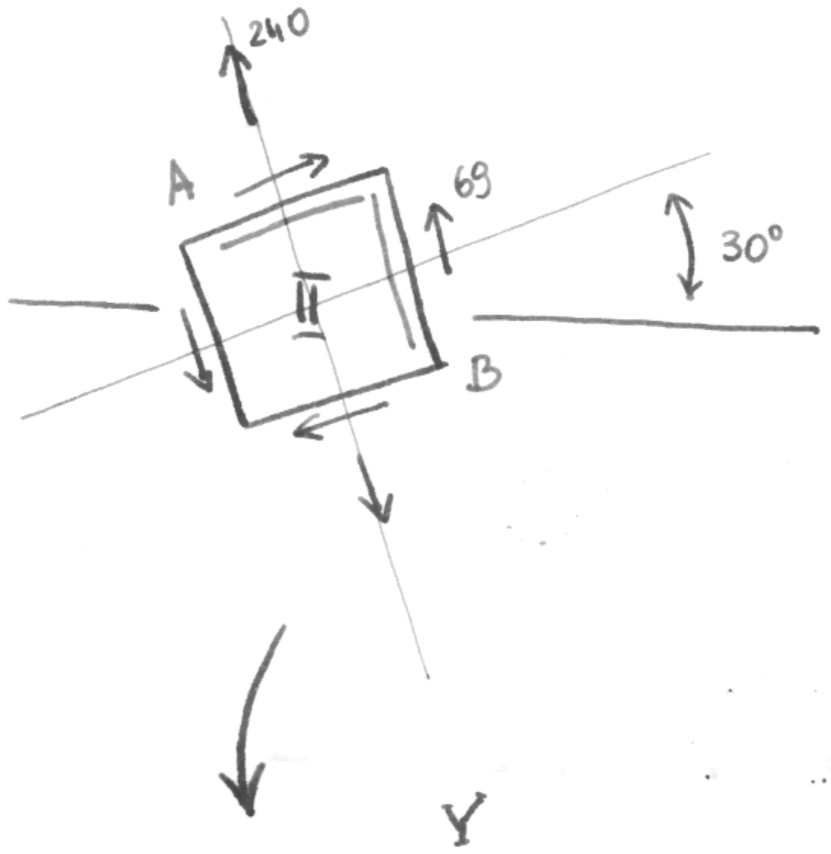
$$\sigma_{mI} = \frac{\sigma_{xI} + \sigma_{yI}}{2} = \frac{60\ MPa - 120\ MPa}{2} = -30\ MPa \qquad \sigma_{mII} = \frac{\sigma_{\eta II} + \sigma_{\xi II}}{2} = \frac{240\ MPa + 0}{2} = 120\ MPa$$

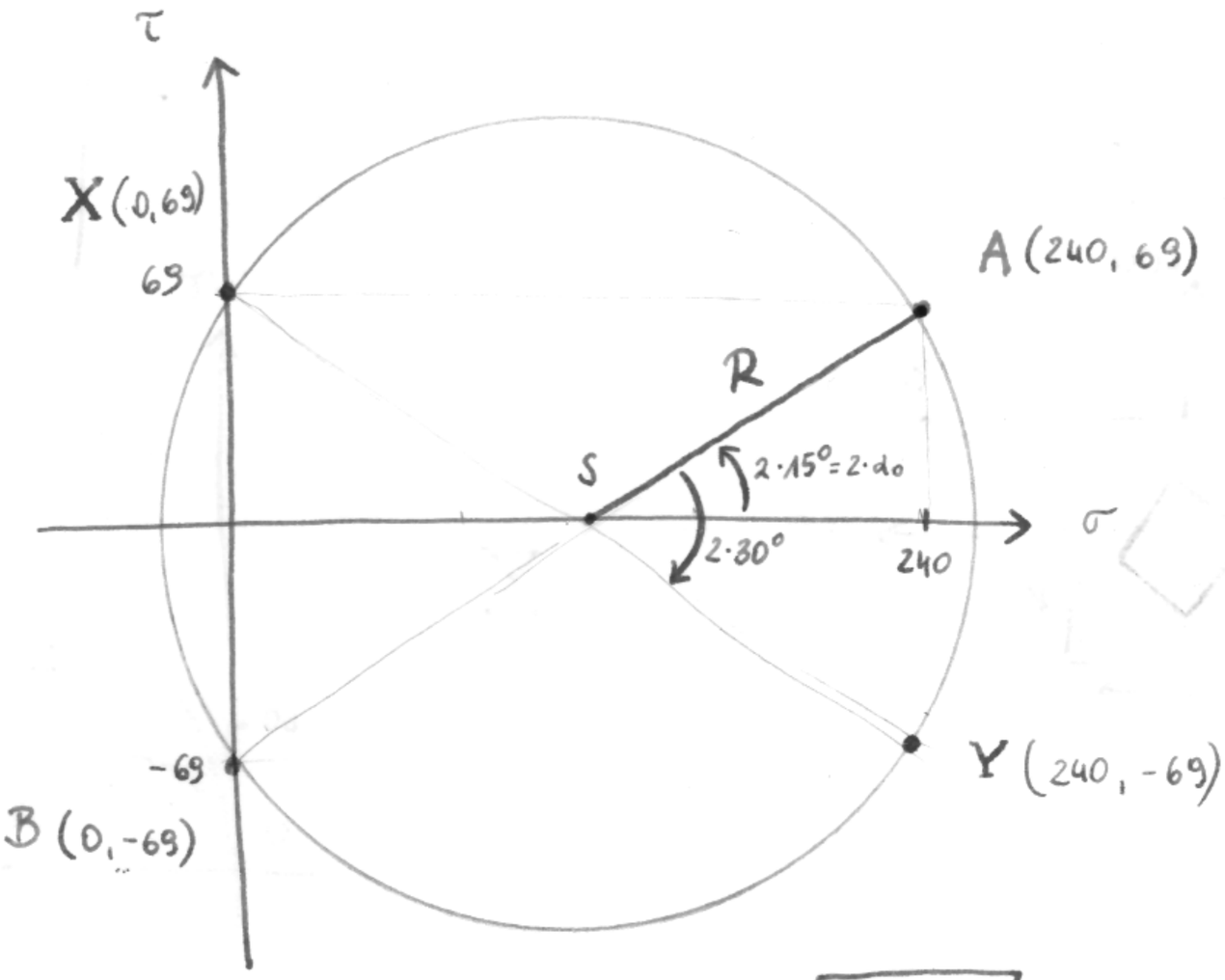
$$R_I = \sqrt{\left(\frac{\sigma_{xI} - \sigma_{yI}}{2}\right)^2 + \tau_{xyI}^2} = \sqrt{\left(\frac{60\ MPa + 120\ MPa}{2}\right)^2 + (-90\ MPa)^2} \approx 127{,}3\ MPa$$

$$\alpha_{0I} = \frac{1}{2}\arcsin\left(\frac{\tau_{yxI}}{R}\right) = 22{,}5°$$

$$\alpha_I = 30° - \alpha_{0I} = 7{,}5°$$

2.

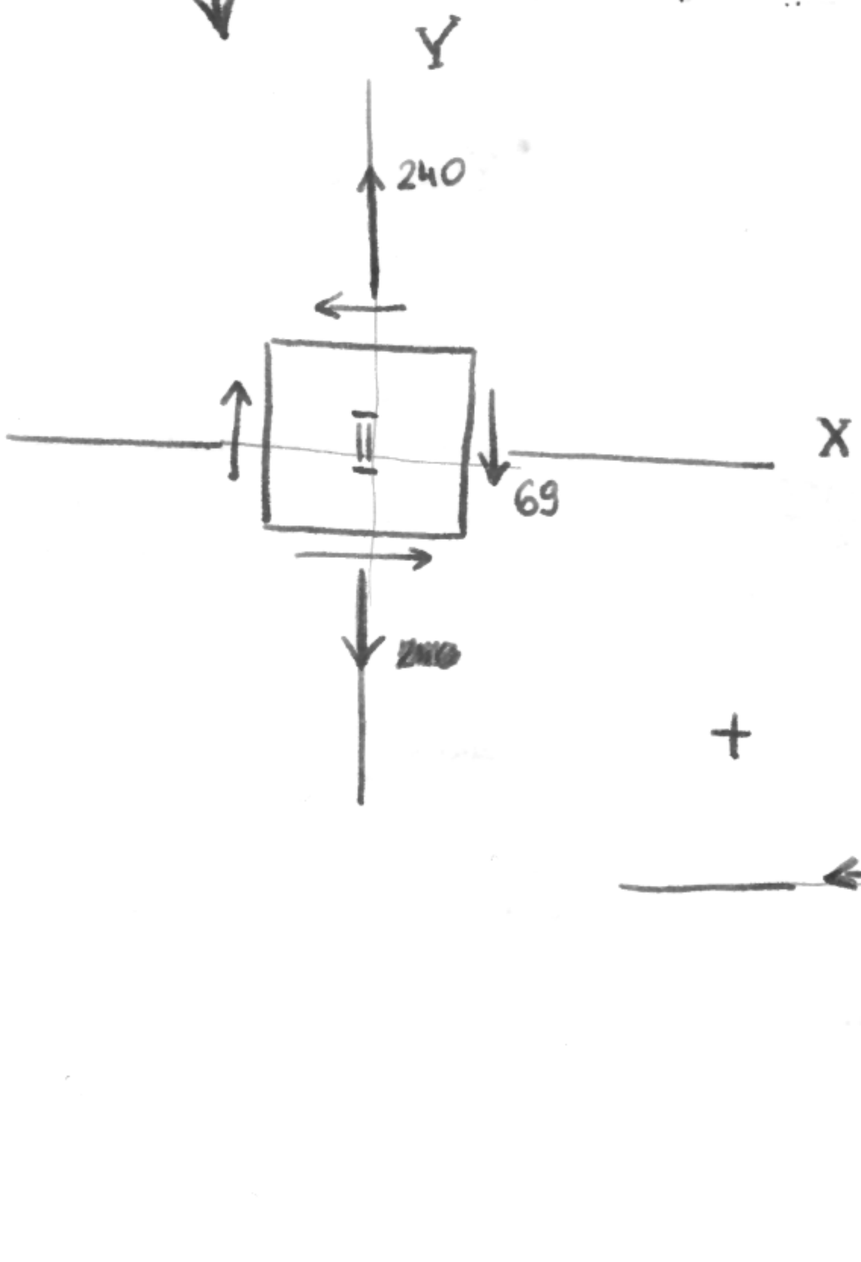




$R = \sqrt{120^2 + 69^2} \approx 138$

$\tan(2\cdot\alpha_0) = \frac{69}{138} = \frac{1}{2}$

$\hookrightarrow \alpha_0 = 15°$



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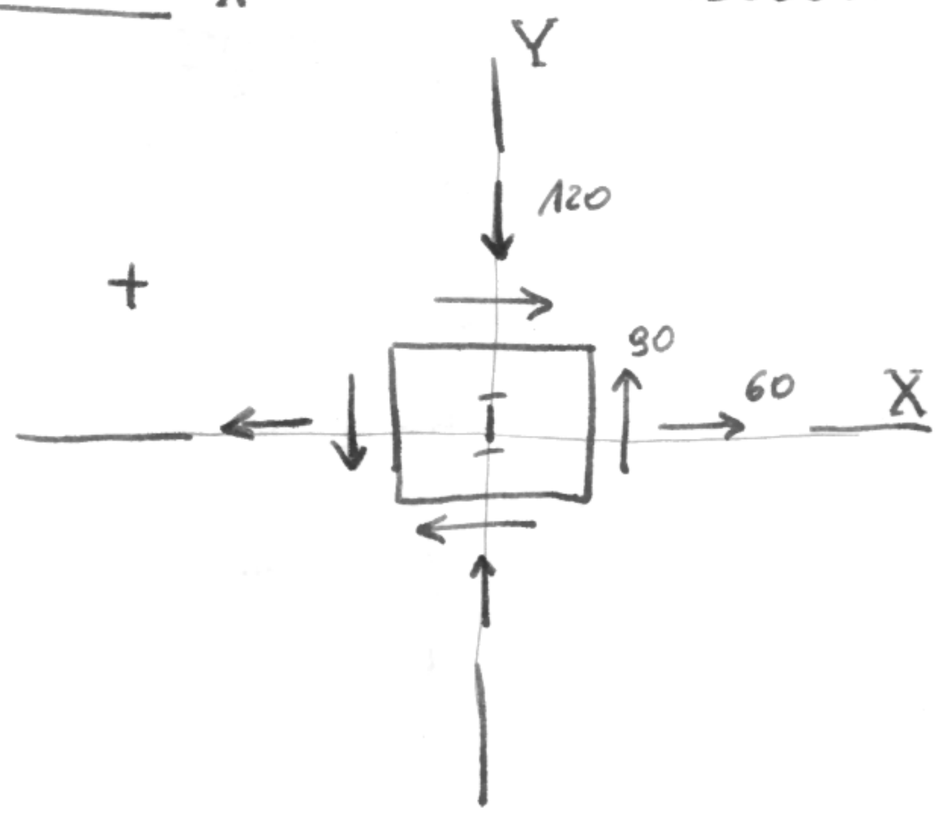

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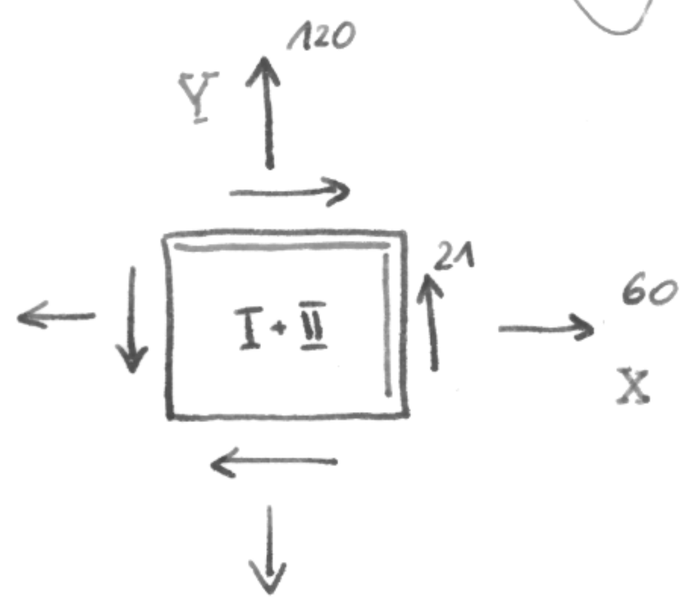
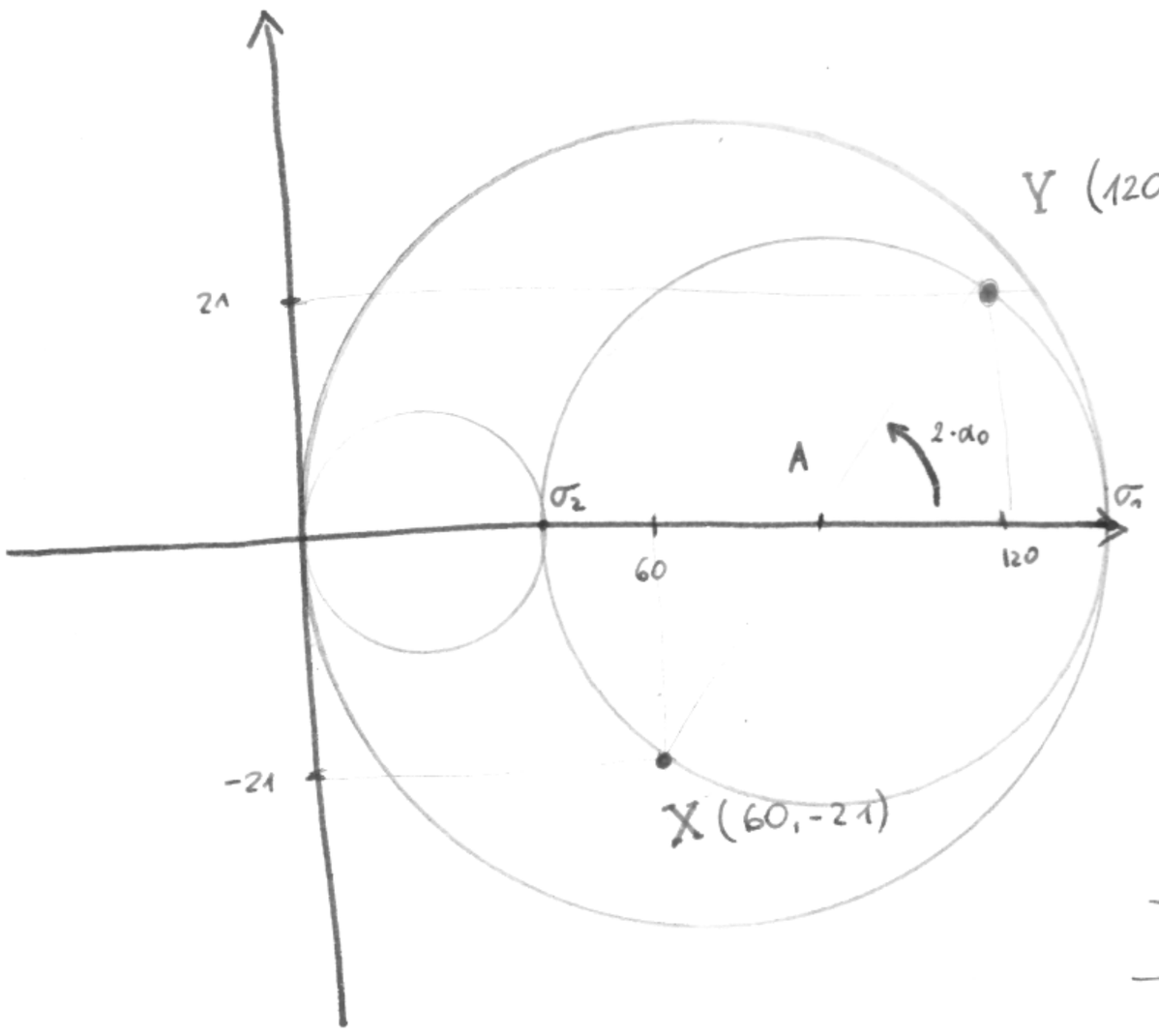


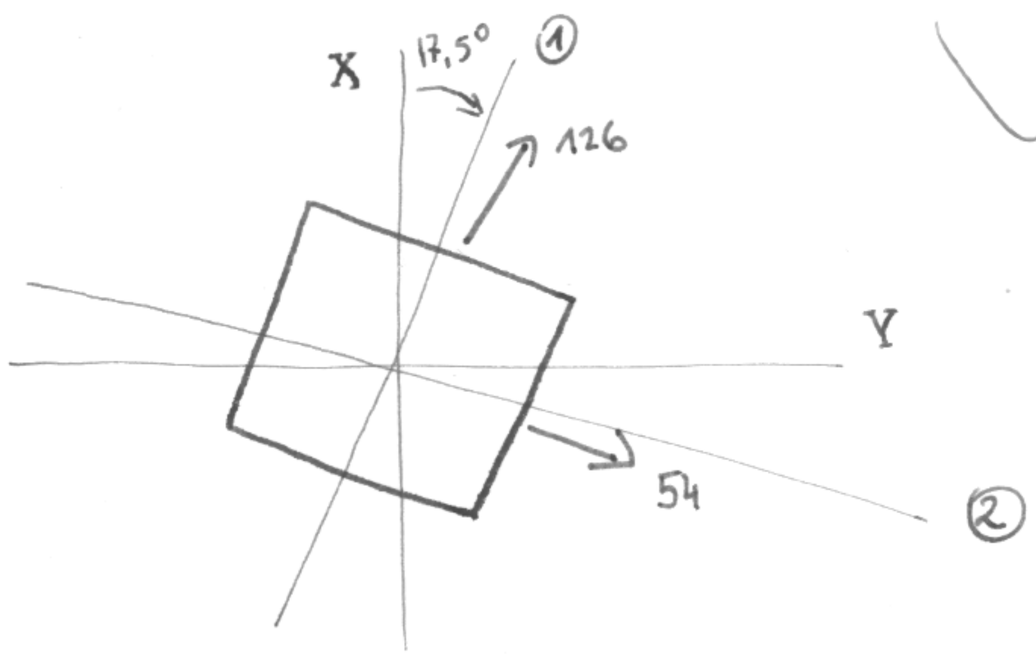


$A = 90 \quad \therefore \quad R = \sqrt{30^2 + 21^2} \approx 36$

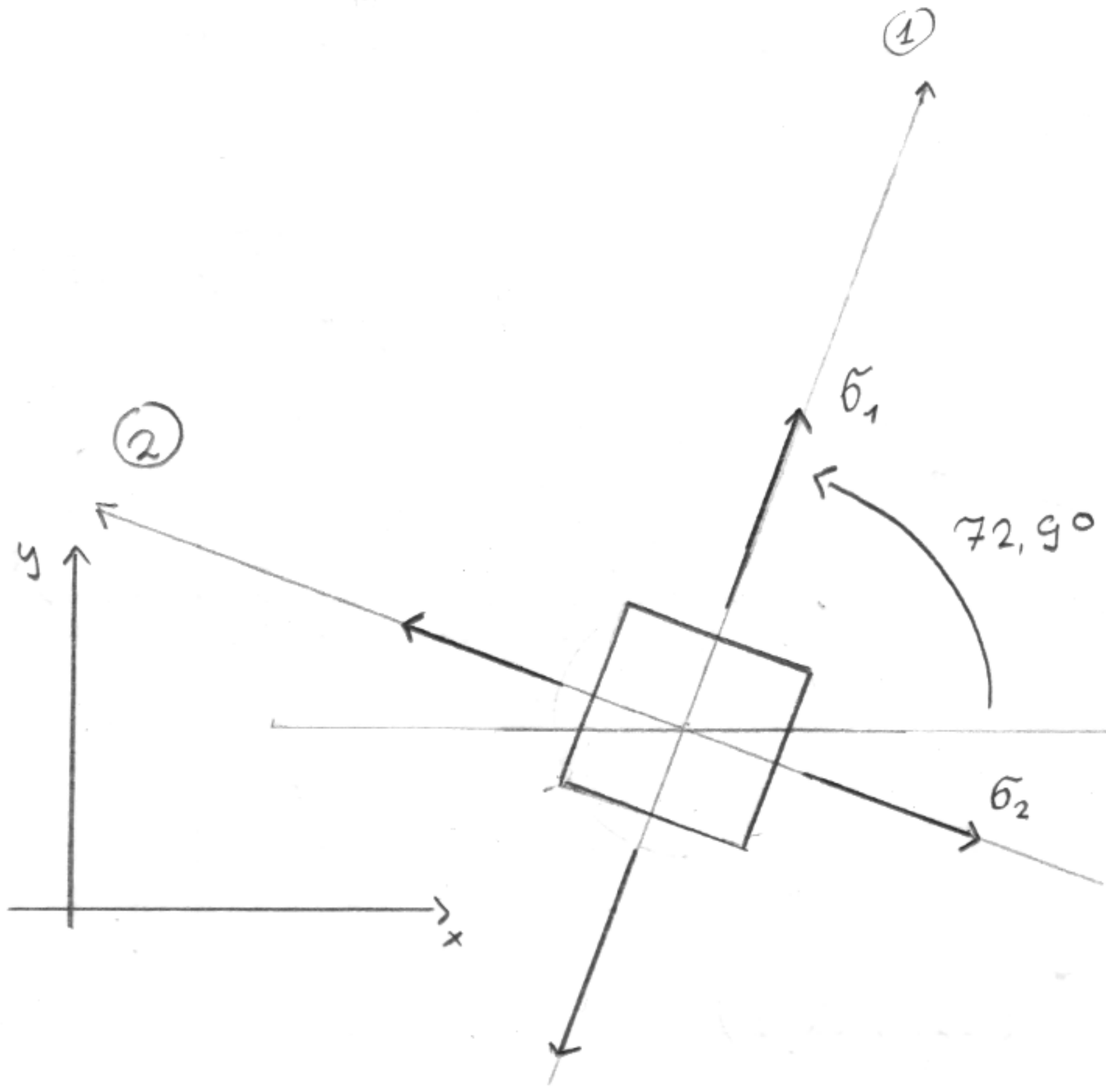
$\begin{cases} \sigma_1 = 126 \\ \sigma_2 = 54 \end{cases}$

$\tan(2\cdot\alpha_0) = \frac{21}{30}$

$\hookrightarrow \alpha_0 = 17{,}5°$



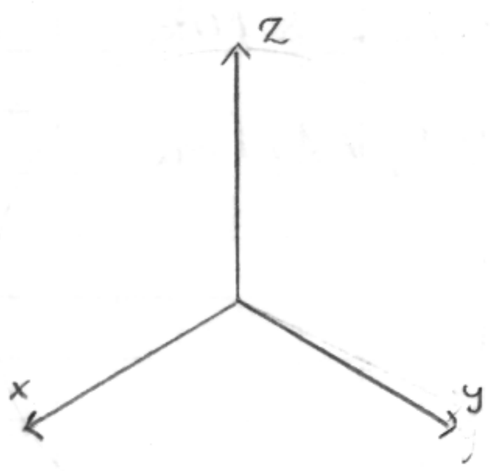
$$\alpha_0 = \frac{1}{2} \arcsin\left(\frac{\tau_{\eta\xi}}{R}\right) \approx 42,9^\circ$$

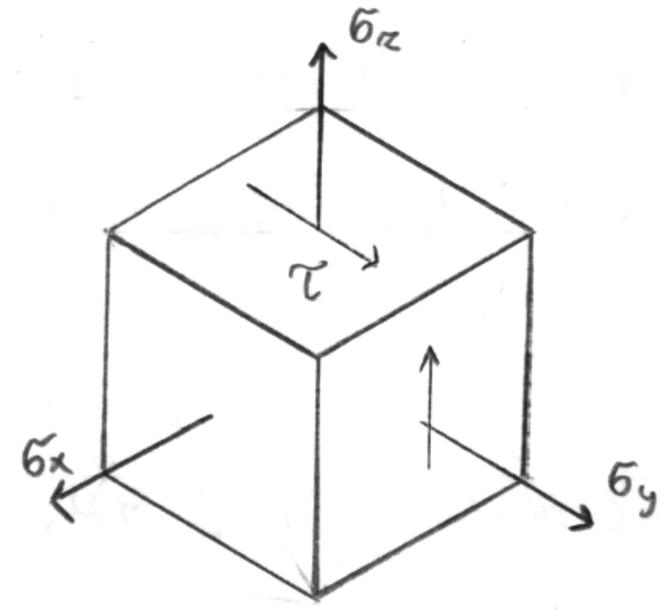


$\sigma_1 = 126,2\ MPa$

$\sigma_2 = 53,8\ MPa$

Zadanie 3:



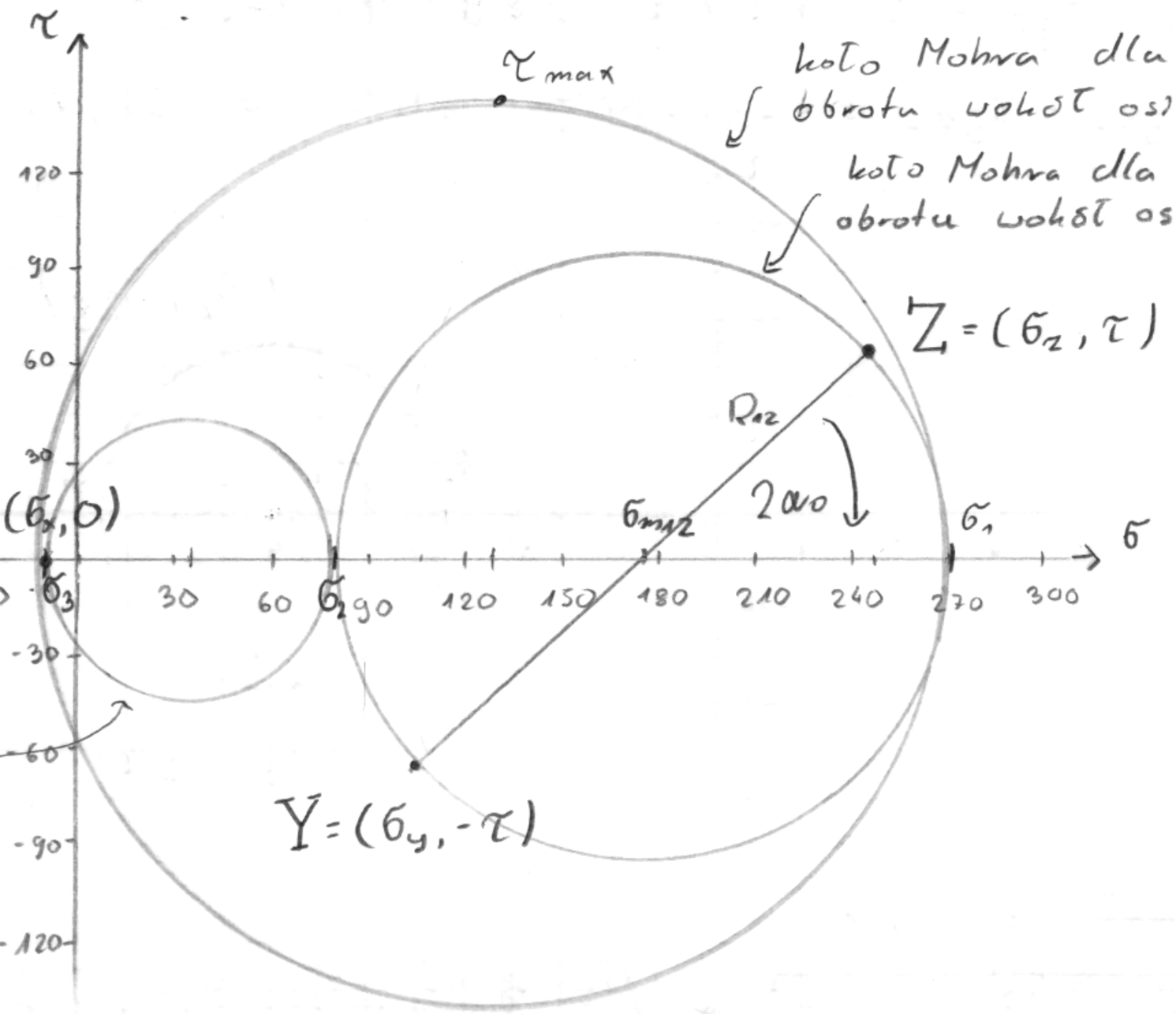


$\sigma_x = -10\ MPa$

$\sigma_y = 106\ MPa$

$\sigma_z = 248\ MPa$

$\tau = 64\ MPa$



$\sigma_x = \sigma_3 = -10\ MPa$

$\sigma_{m12} = \frac{\sigma_2 + \sigma_y}{2} = \frac{248\ MPa + 106\ MPa}{2} = 177\ MPa$

$$R_{12} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{248\,MPa - 106\,MPa}{2}\right)^2 + (64\,MPa)^2} \approx 95,6\,MPa$$

$$\sigma_2 = \sigma_{m12} - R_{12} \approx 81,4\,MPa$$

$$\sigma_1 = \sigma_{m12} + R_{12} \approx 272,6\,MPa$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \approx 141,3\,MPa$$

$$\alpha_0 = \frac{1}{2}\arcsin\left(\frac{-\tau}{R_{12}}\right) \approx -21°$$

$$\sigma_m = \frac{\sigma_1 + \sigma_3}{2} \approx 131,3\,MPa$$